Spatial Dynamic Optimization of Groundwater Use with Ecological Standards for Instream Flow

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Optimization model: maximize farm profits subject to instream flow requirements

- Daily instream flow requirement (ecological goals)

- Hydrologic model: stream-aquifer system where stream depletion effects vary across space and time (Glover-Balmer)

1) Tradeoff between magnitude and duration of stream depletion effect. Optimal allocation of water across wells is differentiated over space and time.

2) In some cases in drought years, wells located closer to the stream should be allocated more water. Duration of the stream depletion effect is more important than the magnitude

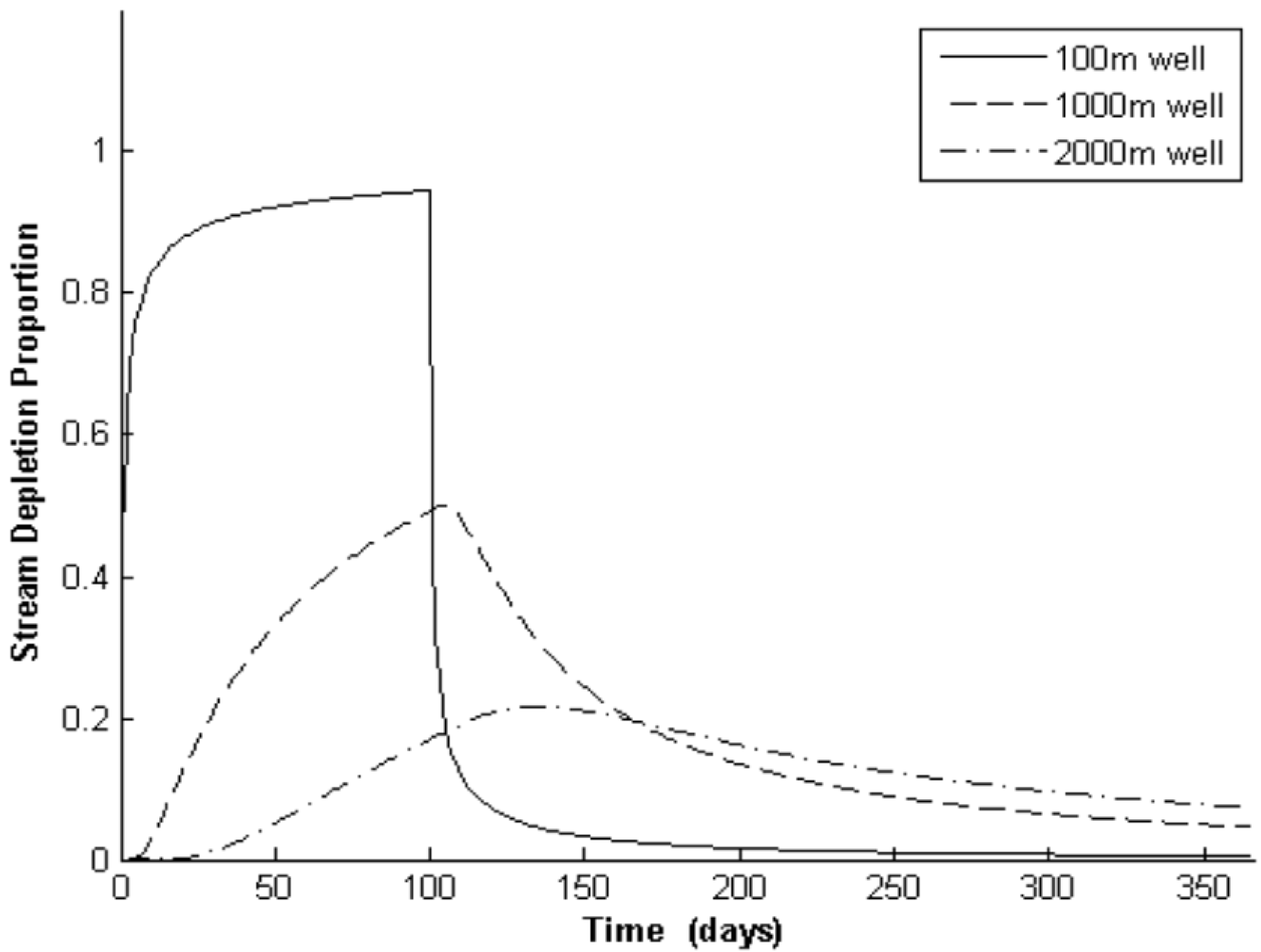
Instream Flow Problem

- Fish habitat quality is dependent in large part on the quantity of streamflow
- Temperature, habitat connectivity, habitat diversity
- Stream depletion in often most critical when need for instream flow is most critical (seasonal rainfall patterns)

Instream Flow Problem

- For protected species, minimum flow standards must be met *every day*
- The timing of stream depletion therefore becomes important (contrast to previous work where magnitude/quantity matters)
- With groundwater, space affects the timing of depletion

Spatial-Dynamic Nature of Stream Depletion Problem



Model

Optimize distribution of daily pumping amounts across a set of irrigation wells

- Least cost policy solution
- Streamflow must meet or exceed minimum threshold on *all days* throughout the irrigation season
- Simulation
  - Two well system:
    - Near (100 m from stream)
    - Distant (500 m from stream)
  - Hold aquifer properties constant

Model

Objective function: maximize sum of individual farm profits (I parcels)

- Profit function with crop yield as a function of applied water on each day of the growing season
- Stream depletion function: Glover-Balmer
- Amount of water drawn from stream is a function of distance of the well from the stream and the time water is removed from the aquifer

Profit Function

Yield increases with water quantity, diminishing returns (consistent with alfalfa)

Deficit irrigation reduces crop yield

- Add a term to the profit function that reduces yield when daily irrigation less than optimum

$$\Gamma_i = \left(\sum_{\tau=1}^{G} \frac{\sum_{g=1}^{\tau} W_{i\tau}}{\sum_{g=1}^{\tau} \frac{U_i^*}{G}}\right)/G$$

Relative yield factor adjustment

- Similar to relative yield in Cai et al 2011

Model

Max Total Farm Profit = $\sum_i^I \Gamma_i * f_i(\Omega_i(G), A_i)$

subject to

- *Daily Water Use:* $0 \le W_{i\tau} \le \frac{U_i^*}{G}$
- *Maximum total yield:* $0 \le \Omega_i(G(W_i)) \le U_i^*$
- *Daily stream depletion:* $\sum_{i=1}^I D_{i\tau} \le F_{i\tau} - \underline{F_{i\tau}}$

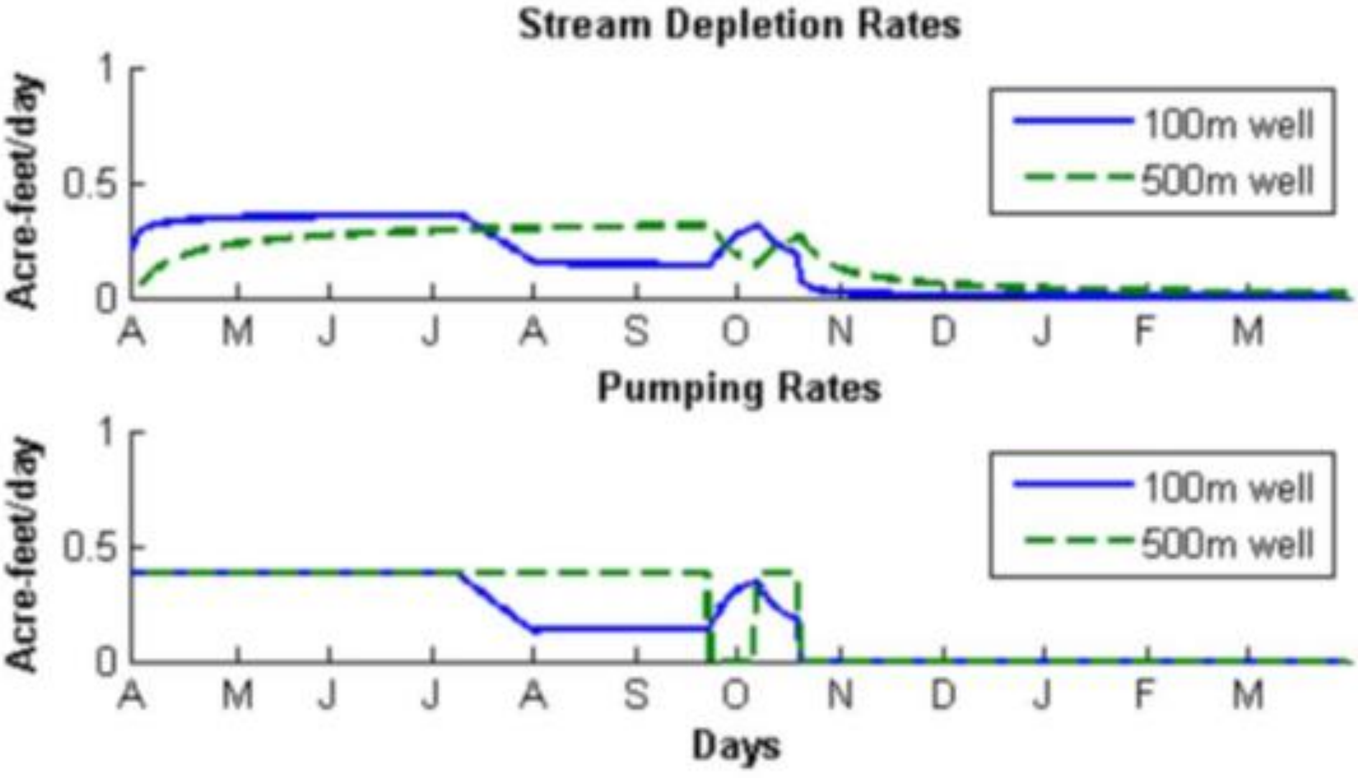
U_i^*/G = optimal daily amount of applied water

Γ_i = relative yield adjustment factor (daily under-watering penalty)

Results

Near and distant wells pump at same rate in early season

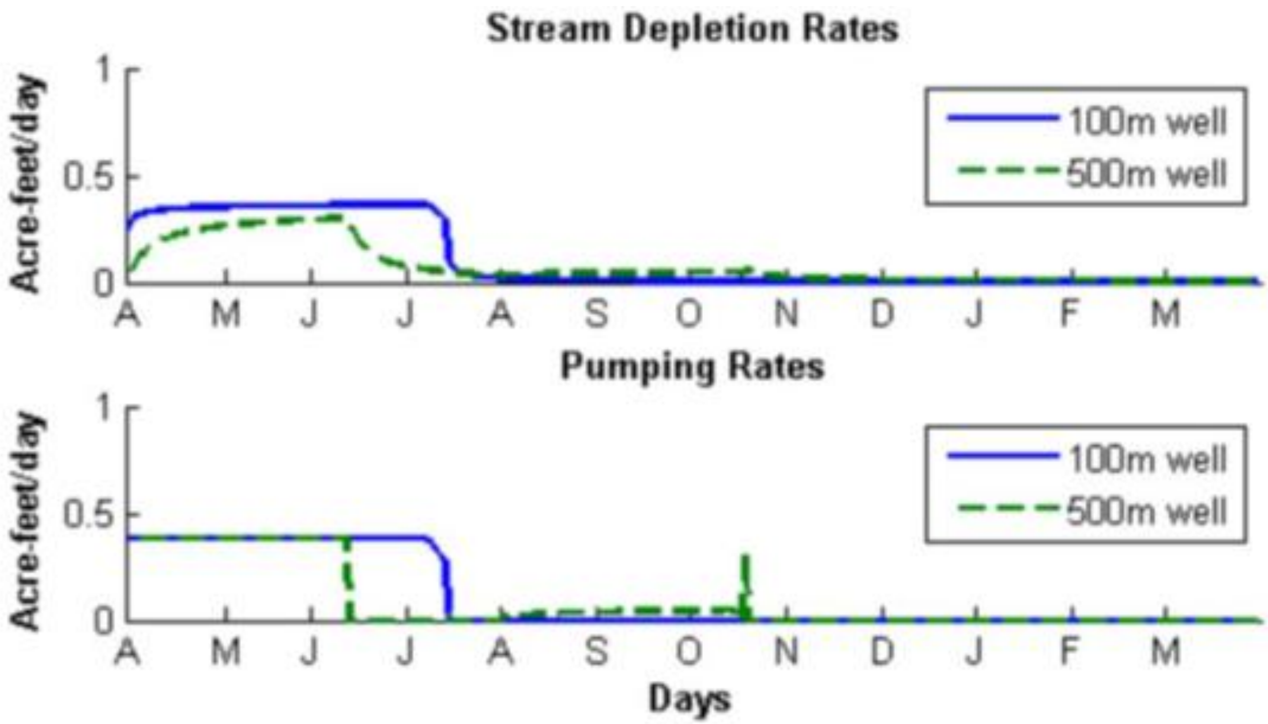
As streamflow declines through the year, pumping at near well is reduced – distant well stream depletion can be distributed over a longer time period



Results (Second Case) "Extreme Low Flow"

Distant wells, even with lower aggregate stream depletion impact, are allocated less water

Persistent residual impact from distant wells – ***Duration of the Stream Depletion Impact***



Conclusion

- There is tradeoff between the magnitude and duration of the stream depletion effect
- Incorporating a time lag in our model reveals that the optimal spatial and temporal distribution of groundwater pumping differs dramatically depending on the level of streamflow
- There exist important cases in drought years where wells located closer to the stream should be allocated more water than wells farther from the stream

Stream Depletion Function

Glover and Balmer 1954

$$D_i = \frac{2W_i}{\sqrt{\pi}} * \int_0^{\infty} \sqrt{\left(\frac{d_i^2 S}{4\tau_i T}\right)} * \ e^{-z^2} dz$$

- *Wi* = pumping rate at well *i*
- *di* = distance from well *i* to the stream
- τ_i = pumping duration
- *S* = Storativity, *T* = Transmissivity

Model (1)

Max Total Farm Profit = $\sum_{i}^{I} f_i(\Omega_i(G), A_i)$

subject to:

1) *Well Capacity:* $0 \leq W_{i\tau} \leq C_i$
2) *Maximum yield:* $0 \leq \Omega_i(G(W_i)) \leq U_i^*$
3) *Stream Depletion:* $\sum_{i=1}^{I} D_{i\tau} \leq F_{i\tau} - \underline{F_{i\tau}}$

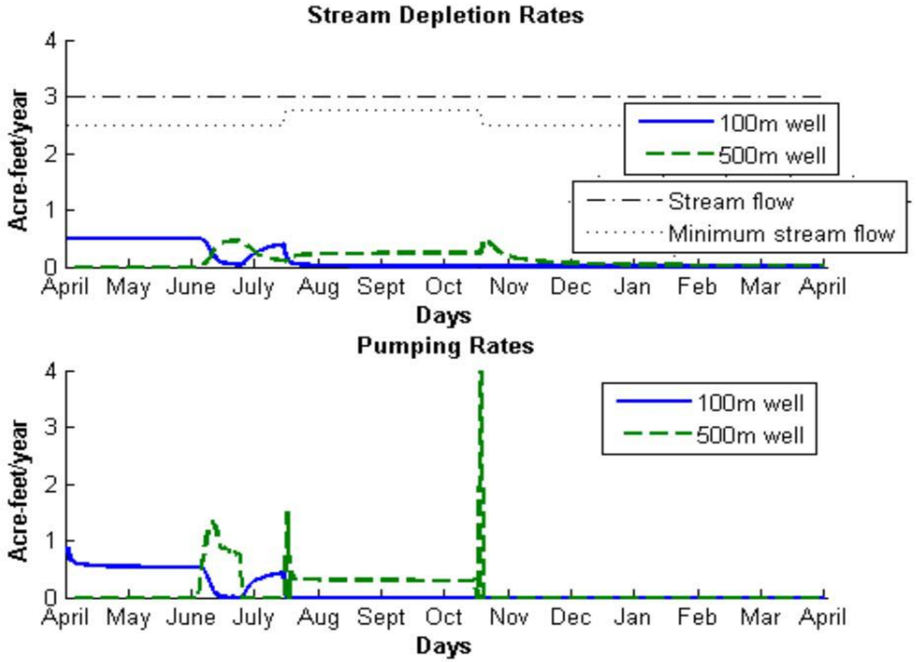
$W_{i\tau}$ = *quantity of water pumped by well i at time* τ

Yield is independent of timing of water application

Results (First Case)

1. Annual water allocation greater for distant wells – total stream depletion is less for these wells
2. Irrigation season subdivided into two

- Near wells = early season, Distant wells = late season
- Residual impact from distant wells reduces amount available for near wells



Profit Function

- Yield increases with water quantity, diminishing returns (consistent with alfalfa)

$$\frac{\partial f_i}{\partial \Omega_i} \geq 0, \frac{\partial^2 f_i}{\partial \Omega_i^2} \leq 0$$

$$profit_i = \sum_{\tau=1}^{G} MRP * W_{i\tau} - 0.5 * \frac{MRP}{U_i^*} * W_{i\tau}^2$$

G = number of days in the growing season

$\Omega_i(G)$ = total quantity of water pumped by well I

MRP = marginal revenue product of water

Ui = maximum of*